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IMPURITY DISPERSION IN NONUNIFORM FLOWS

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The theory of impurity dispersion in tubes owes its origin to the work of Taylor [1], where the equation of diffusion was derived with constant coefficients for the impurity concentration averaged over the cross section, this equation replacing the local diffusion equation with convective terms dependent on the coordinate transverse to the flow. Aris [2] subsequently refined the value of the coefficient of effective diffusion (dispersion), he proposed formulas for the coefficient of dispersion in a tube with an arbitrary lateral cross section, and he refined the areas of applicability for the Taylor theory as one which is asymptotic for sufficiently long times. At the present time, the theory of dispersion is covered in a large number of studies. We can cite a number of these, containing unique approaches to the problem [3-5] and refining the Taylor–Aris model for lower time values, and then [6, 7], developing the theory in various directions.

A significant feature of the above-cited and similar studies on the theory of dispersion is the uniformity of fluid motion and the independence of the velocity component relative to the longitudinal coordinate, thus limiting the applicability of the theory, essentially, to prismatic tubes. Whereas in nature and in technology one frequently encounters nonuniform flows that are, in a certain sense, similar to the flow in tubes, in order to describe the propagation of impurities through these tubes it would be desirable to derive the equations of equivalent diffusion, analogous to the Taylor dispersion equation. The solution of this problem for a number of cases is precisely the aim of this study.

1. Dispersion of Impurities in Elongated Zones, in the Nonuniform Flow of a Fluid. Elongation of a zone indicates that one of the measurements of the flow region considerably exceeds the two remaining measurements, and it is the propagation of the impurity precisely in that direction that is of interest to us. Flows of this kind are formed in the stagnation regions of various pieces of equipment, in tubes when barriers are present, etc. Here and below, we will conduct our study with a coordinate system that is either nonmoving or moving at some mean velocity, as well as for cases in which there is no mean motion in any of the directions we have selected. We will assume the flow of the fluid to be laminar, and the fluid itself to be incompressible. Under these assumptions, the equation of convective diffusion in dimensionless coordinates has the form

$$\varepsilon^2 \frac{\partial c}{\partial t} + \varepsilon \,\mathrm{Pe}\left(u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z}\right) = \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \varepsilon^2 \frac{\partial^2 c}{\partial z^2}, \tag{1.1}$$

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where $x = X/a$, $y = Y/a$, $z = Z/\ell$, $\varepsilon = a/\ell$, $\mathrm{Pe} = aw^*/D$, $t = D\tau/\ell^2$. Here X, Y, Z, and τ are the dimensional coordinates and time; a is the characteristic dimension of the region in the X, Y plane; ℓ is the characteristic dimension in the direction of the Z axis; D is the coefficient of molecular diffusion; w^* is the scale of velocity in the Z direction, which is connected to the scale v^* in the X- and Y-directions by the relationship $v^*\ell = w^*a$, following out of the equation of continuity

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0. \tag{1.2}$$

We will assume that $\varepsilon \ll 1$. The choice of the orders of magnitude in the terms of Eq. (1.1) may prove to be somewhat artificial; however, it corresponds to the Taylor procedure of successive approximations [1]. Later on we will prove that utilization of the methods from the theory of perturbations [8, 9] will bring us, in this particular case, to the Taylor and Aris results. Use of the method of perturbations serves as a more flexible tool by means of which, in the case of need, we can find correction factors for dispersion equations of the Taylor—Aris type. Apparently, the results obtained by the method of perturbations in such problems exhibits a more common significance (not only in the case of small ε). This is a result of the fact that the theory of dispersion can be used in the case of sufficiently long periods of time, and it is probable that certain of the scales of length that entered into the formulation of the problem to be characteristic, owing to the diffusion i.e., "spreading out" of the initially specified concentration profiles, etc. At the same time, the asymptotic nature of the theory of dispersion has been studied in detail [2, 3, etc.], and the derivation of the identical results through the method of perturbations and through methods containing no small parameters may serve as some justification of the reliability of our choice for the small parameter and for the possibility of extending these results into the region of larger ε.

Equation (1.1) should be enhanced with the initial and boundary conditions, from which the following are of significance to us:

$$\partial c/\partial n|_\gamma = 0,\ \gamma = \partial\Omega\cdot R_1; \tag{1.3}$$

$$c|_{t=0} = F(x, y, z). \tag{1.4}$$

We will look for the solution of problem (1.1)-(1.4) in the form of the expansion $c = c_0 + \varepsilon c_1 + \varepsilon^2 c_2 + \ldots$, which on substitution into Eq. (1.1) and into boundary condition (1.3) will give us the following sequence of problems:

$$\Delta c_0 = 0,\ \partial c_0/\partial n|_\gamma = 0; \tag{1.5}$$

$$\Delta c_1 = \mathrm{Pe}\left[u\frac{\partial c_0}{\partial x} + v\frac{\partial c_0}{\partial y} + w\frac{\partial c_0}{\partial z}\right],\ \left.\frac{\partial c_1}{\partial n}\right|_\gamma = 0; \tag{1.6}$$

$$\begin{gathered}\Delta c_i = \mathrm{Pe}[u\partial c_{i-1}/\partial x + v\partial c_{i-1}/\partial y + w\partial c_{i-1}/\partial z] - \\ - \partial^2 c_{i-2}/\partial z^2 + \partial c_{i-2}/\partial t,\ \partial c_i/\partial n|_\gamma = 0,\ i = 2, 3, \ldots,\end{gathered} \tag{1.7}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. We should note that there exist no time derivatives of the unknown functions in the equations of problems (1.5)-(1.7), which tells us that the perturbations are singular in nature and that for complete analysis we have to resort to a finer time scale, namely internal time [8, 9].

The solution of problem (1.5) will be the as yet unknown function of z and t, i.e., $c_0 = c_0(z, t)$. Then, using the Green's function G for the Neumann problem (1.6), we can write its solution in the form [10]

$$c_1 = \mathrm{Pe}\frac{\partial c_0}{\partial z}\int\limits_\Omega w(\xi, \eta, z)\, G(x, y, \xi, \eta)\, d\xi\, d\eta + c_1^*(z, t) \tag{1.8}$$

(c_1^* is an as-yet indeterminate function of z and t). We will now integrate Eq. (1.7) over the section Ω for i = 2. Using (1.8), the Green's theorem, the continuity equation (1.2), and the conditions of impenetrability of u, v = 0 on the γ contour of the region in the x, y plane, we will obtain

$$\partial c_0/\partial t = \partial\{[1 + \mathrm{Pe}^2 D^*(z)]\partial c_0/\partial z\}/\partial z, \tag{1.9}$$

which is the dispersion equation that serves to generalize the Taylor and Aris relationships. Here

$$D^* = -\frac{1}{s}\int\limits_{\Omega} w(x, y, z)\,dx\,dy \int\limits_{\Omega} w(\xi, \eta, z)\,G(x, y, \xi, \eta)\,d\xi\,d\eta \tag{1.10}$$

is the coefficient of equivalent diffusion. In the particular case of flows in which w is independent of z, formulas (1.9) and (1.10) coincide with the Taylor–Aris formulas (we did not take into consideration the possible relationship between the coefficient of diffusion and the x and y coordinates, hypothesized in [2] and of no fundamental significance in the case of our method), and this is easily proved by turning to dimensionless coordinates.

Let us represent the Green's function in the form of the sum

$$G(x, y, \xi, \eta) = -\sum_{i=1}^{\infty} \frac{\mathscr{Y}_i(x, y)\,\mathscr{Y}_i(\xi, \eta)}{\lambda_i},$$

where $\mathscr{Y}_i$ are the normalized eigenfunctions of the Neumann problem in region Ω, satisfying equation $\Delta\mathscr{Y}_i + \lambda_i\mathscr{Y}_i = 0$, $i = 1, 2, \ldots$, while the eigenvalues λ_i of the problem are positive, and we thus come to the expression

$$D^* = \frac{1}{s}\sum_{i=1}^{\infty} \frac{1}{\lambda_i}\left[\int\limits_{\Omega} w(x, y, z)\,\mathscr{Y}_i(x, y)\,dx\,dy\right]^2, \tag{1.11}$$

from which the positiveness of D^* is obvious and which can be utilized to calculate or estimate D^* by various methods.

A significant difference in formulas (1.10) and (1.11) from those conventionally used in the nonsteady equations of the theory of dispersion is the relationship between D^* and the z coordinate. This is particularly important for large values of the Pe number, which is typical of dispersion problems when the coefficient D is considerably smaller than the "convective" part $a^2w^{*2}D^*/D$ of the dispersion factor, virtually throughout the entire region of flow, with the exception of the surroundings about those points z at which w in conjunction with D^* vanish. This latter situation may arise at solid surfaces where w exhibits a zero value of second order, while D^* has a zero fourth-order value, and also on liquid surfaces separating circulation zones, where D^* exhibits a second-order zero value. These circumstances allow us to construct local (boundary-layer) equations which, when made appropriately dimensionless, can be written in the form

$$\partial c_0/\partial t = \partial[(1 + z^4)\partial c_0/\partial z]/\partial z,$$
$$\partial c_0/\partial t = \partial[(1 + z^2)\partial c_0/\partial z]/\partial z.$$

These equations, describing the exchange of mass near the points where w = 0, in a known time scale frequently determine the progress of the entire process, since in the remaining portion of the z region concentration is rapidly evened out (when Pe ≫ 1). We note that the second equation can be used to calculate the exchange of mass in closed cavities streamlined with an external flow in accordance with the Lavrent'ev scheme [11, 12].

The initial condition for Eq. (1.9) is obtained by means of the joining procedure with an internal solution [8, 9] describing the process for limited times. Let us introduce the "internal" time $\zeta = t/\varepsilon^2$. For the function of the zeroth approximation of the internal solution with respect to ε, after passing to the limit $\varepsilon \to 0$, we derive the equation

$$\partial c^\circ/\partial\zeta = \Delta c^\circ, \tag{1.12}$$

where the degree sign marks the internal solution. Having integrated Eq. (1.12) over the entire region, utilizing the Green's theorem, resorting to boundary conditions (1.3), and then having integrated over ζ with consideration of the initial condition (1.4), we find

$$\int_{\Omega} c^{\circ}\, dx\, dy = \int_{\Omega} F(x, y, z)\, dx\, dy. \tag{1.13}$$

Now, passing to the limit $\zeta \to \infty$ in Eq. (1.13) and using the principle of limit joining $c^{\circ}(x, y, z, \infty) = c_0(z, 0)$, we obtain

$$c_0 |_{t=0} = \langle F \rangle = \frac{1}{s} \int_{\Omega} F(x, y, z)\, dx\, dy \tag{1.14}$$

as the unknown initial condition for Eq. (1.9); s is the dimensionless area (referred to a^2) of the transverse cross section of the channel. Extending the described procedure, we can find the equation for the function c_1 (more precisely, c_1^*), analogous to (1.9), but with source terms, and a pertinent initial condition.

2. Dispersion of the Impurity in Thin Liquid Films. Let us examine the general case of dispersion in some arbitrary orthogonal coordinate system x_1, x_2, x_3, where the extent of the region in the x_3 direction is considerably smaller than in the x_1 and x_2 directions. The equation of convective diffusion in dimensionless variables is written in the form

$$\nu^2 H_1 H_2 H_3 \frac{\partial c}{\partial T} + \nu \,\mathrm{Pe} \left[u_1 H_2 H_3 \frac{\partial c}{\partial x_1} + u_2 H_1 H_3 \frac{\partial c}{\partial x_2} + u_3 H_1 H_2 \times \right.$$
$$\left. \times \frac{\partial c}{\partial x_3} \right] = \frac{\partial}{\partial x_3} \left(\frac{H_1 H_2}{H_3} \frac{\partial c}{\partial x_3} \right) + \nu^2 \left[\frac{\partial}{\partial x_1} \left(\frac{H_2 H_3}{H_1} \frac{\partial c}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\frac{H_1 H_3}{H_2} \frac{\partial c}{\partial x_2} \right) \right], \tag{2.1}$$

where $x_1 = X_1/a$; $x_2 = X_2/a$; $x_3 = X_3/\ell$; $\nu = \ell/a$; $\mathrm{Pe} = v^* \ell/D$; $T = D\tau/a^2$. The velocity components satisfy the continuity equation

$$\partial(u_1 H_2 H_3)/\partial x_1 + \partial(u_2 H_1 H_3)/\partial x_2 + \partial(u_3 H_1 H_2)/\partial x_3 = 0,$$

which, as before, establishes the relationship between the scale of the velocity v^* along the X_1 and X_2 axes with the velocity scale along the X_3 axis. Here H_1, H_2, and H_3 are the Lamé coefficients which we assume to be such that there exist the H_i containing integrals which appear below. We will assume the thickness of the film to be equal to ℓ, so that $x_3 \in (0, 1)$. We will enhance Eq. (2.1) with the following conditions:

$$\partial c/\partial x_3 |_{x_3=0;1} = 0; \tag{2.2}$$

$$c|_{T=0} = R(x_1, x_2, x_3). \tag{2.3}$$

Other conditions necessary for complete formulation of the problem, as before, are inconsequential for our analysis. We will assume ν to be a small parameter and we will seek the solution by the method of perturbations in the form of a series over the powers of ν. We derive the following chain of equations:

$$\frac{\partial}{\partial x_3} \left(\frac{H_1 H_2}{H_3} \frac{\partial c_0}{\partial x_3} \right) = 0; \tag{2.4}$$

$$\frac{\partial}{\partial x_3} \left(\frac{H_1 H_2}{H_3} \frac{\partial c_1}{\partial x_3} \right) = \mathrm{Pe} \left[u_1 H_2 H_3 \frac{\partial c_0}{\partial x_1} + u_2 H_1 H_3 \frac{\partial c_0}{\partial x_2} + u_3 H_1 H_2 \frac{\partial c_0}{\partial x_3} \right]; \tag{2.5}$$

$$\frac{\partial}{\partial x_3} \left(\frac{H_1 H_2}{H_3} \frac{\partial c_i}{\partial x_3} \right) = \mathrm{Pe} \left[u_1 H_2 H_3 \frac{\partial c_{i-1}}{\partial x_1} + u_2 H_1 H_3 \frac{\partial c_{i-1}}{\partial x_2} + u_3 H_1 H_2 \frac{\partial c_{i-1}}{\partial x_3} \right] +$$
$$+ H_1 H_2 H_3 \frac{\partial c_{i-2}}{\partial T} - \frac{\partial}{\partial x_1} \left(\frac{H_2 H_3}{H_1} \frac{\partial c_{i-2}}{\partial x_1} \right) - \frac{\partial}{\partial x_2} \left(\frac{H_1 H_3}{H_2} \frac{\partial c_{i-2}}{\partial x_2} \right), \quad i = 2, 3, \ldots, \tag{2.6}$$

where the subscript identifies the approximation number. Each of these equations is subject to conditions (2.2) with the appropriate subscript for c. From (2.4), with consideration of (2.2), we find $c_0 = c_0(x_1, x_2, T)$. Then, from (2.5), after integration, we obtain

$$c_1 = \mathrm{Pe}\left[\frac{\partial c_0}{\partial x_1}\int\limits_0^{x_3}\frac{H_3\,dz}{H_1H_2}\int\limits_0^{z} u_1H_2H_3\,d\xi + \frac{\partial c_0}{\partial x_2}\int\limits_0^{x_3}\frac{H_3\,dz}{H_1H_2}\int\limits_0^{z} u_2H_1H_3\,d\xi\right] + \tag{2.7}$$
$$+\, c_1^*(x_1, x_2, T),$$

where the function c_1^* can be determined by continuing the described procedure. Let us integrate Eq. (2.6) for the case in which i = 2 over x_3 within the limit (0, 1). When we use (2.7), the continuity equation, and the condition of impenetrability of the planes z = 0 and z = 1 for the liquid, we find the equation

$$\begin{aligned} m(x_1, x_2)\frac{\partial c_0}{\partial T} &= \frac{\partial}{\partial x_1}\left[D_1(x_1, x_2)\frac{\partial c_0}{\partial x_1}\right] + \frac{\partial}{\partial x_2}\left[D_2(x_1, x_2)\frac{\partial c_0}{\partial x_2}\right] + \\ &+ \mathrm{Pe}^2\left\{\frac{\partial}{\partial x_1}\left[D_{11}(x_1, x_2)\frac{\partial c_0}{\partial x_1} + D_{12}(x_1, x_2)\frac{\partial c_0}{\partial x_2}\right] + \right. \\ &\left. + \frac{\partial}{\partial x_2}\left[D_{21}(x_1, x_2)\frac{\partial c_0}{\partial x_1} + D_{22}(x_1, x_2)\frac{\partial c_0}{\partial x_2}\right]\right\}, \end{aligned} \tag{2.8}$$

where

$$\begin{gathered} D_1(x_1, x_2) = \int\limits_0^1 \frac{H_2H_3}{H_1}\,dx_3, \quad D_2(x_1, x_2) = \int\limits_0^1 \frac{H_1H_3}{H_2}\,dx_3, \\ m(x_1, x_2) = \int\limits_0^1 H_1H_2H_3\,dx_3, \quad \Phi(x_1, x_2, \alpha) = \int\limits_0^{\alpha}\frac{H_3\,dx_3}{H_1H_2}, \end{gathered} \tag{2.9}$$

$$D_{11}(x_1, x_2) = -\frac{1}{2}\int\limits_0^1\int\limits_0^1 |\Phi(\alpha) - \Phi(\beta)|\, u_1(\alpha)H_2(\alpha)H_3(\alpha)u_1(\beta)H_2(\beta)H_3(\beta)\,d\alpha\,d\beta,$$

$$D_{12}(x_1, x_2) = D_{21}(x_1, x_2) =$$
$$= -\frac{1}{2}\int\limits_0^1\int\limits_0^1 |\Phi(\alpha) - \Phi(\beta)|\, u_1(\alpha)H_2(\alpha)H_3(\alpha)u_2(\beta)H_1(\beta)H_3(\beta)\,d\alpha\,d\beta,$$

$$D_{22}(x_1, x_2) =$$
$$= -\frac{1}{2}\int\limits_0^1\int\limits_0^1 |\Phi(\alpha) - \Phi(\beta)|\, u_2(\alpha)H_1(\alpha)H_3(\alpha)u_2(\beta)H_1(\beta)H_3(\beta)\,d\alpha\,d\beta.$$

In formulas (2.9) for D_{ij} beneath the integral sign we have dropped the parameters x_1 and x_2, leaving only the integration variable. In the derivation of these formulas it was the following conditions essentially that were used:

$$\int\limits_0^1 u_1H_2H_3\,dx_3 = \int\limits_0^1 u_2H_1H_3\,dx_3 = 0$$

which represent the absence of an average transport of impurities in the directions of the x_1 and x_2 axes. If the "diffusion" coefficients D_1 and D_2, which are simply averaged combinations of the Lamé coefficients, i.e., they have a purely geometric origin, then in the expressions for D_{ij} we find velocity components and it is precisely these coefficients that are analogous to the Taylor coefficients [1]. The nonnegativeness of the coefficients D_{11} and D_{22} follows out of another form of their notation and from the conditions $H_i \geq 0$, i = 1, 2, 3:

$$D_{11} = \int\limits_0^1 \frac{H_3\,dx_3}{H_1H_2}\left[\int\limits_0^{x_3} u_1H_2H_3\,dx_3\right]^2 \geqslant 0. \tag{2.10}$$

We also have an analogous relationship for D_{22}. It is interesting to note that the coefficients D_{ij} in combination with (2.10) also satisfy the inequality

$$D_{11}D_{22} \geqslant D_{12}^2, \tag{2.11}$$

which in linear nonequilibrium thermodynamics follows out of the condition of positive definiteness for the establishment of entropy [13] in fulfilling the reciprocity relationship $D_{12} = D_{21}$ for the kinetic coefficient. To prove (2.11) it is sufficient in the place of $u_1H_2H_3$ beneath the sign of the internal integral in (2.10) to substitute $u_1H_2H_3 + pu_2H_1H_3$, and since the resulting expression that is quadratic with respect to p must be nonnegative for any p, its discriminant must be nonpositive, which will then lead, after certain transformations, to inequality (2.11).

As before, in Sec. 1, with large values for the Pe number, of some significance are the lines in the x_1, x_2 plane, where the velocity components u_1, u_2 vanish. Once again it becomes possible to construct the boundary-layer equations as functions of the zero orders of the functions u_1 and u_2. In analogy with Sec. 1, it is also proved that the initial condition for Eq. (2.8) will be

$$c_0|_{T=0} = \langle R\rangle = \frac{1}{m}\int\limits_0^1 R(x_1, x_2, x_3)H_1H_2H_3\,dx_3.$$

Let us note that in our consideration of the specific coordinate systems in certain expressions containing the Lamé coefficients the introduced parameter ν appears in natural fashion and the corresponding formulas can be simplified as $\nu \to 0$. We have not done this, desirous of preserving the ability to interpret Eq. (2.8) as one that is asymptotic as $T \to \infty$ and with the nonobligatory smallness of ν (see the remark in Sec. 1). Let us also note that for the coefficients D_{ij} we can present formulas analogous to (1.11). These have a particularly simple form in the Cartesian coordinate system

$$D_{11} = \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{\alpha_j^2}{j^2}, \quad D_{12} = \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{\alpha_j\beta_j}{j^2}, \quad D_{22} = \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{\beta_j^2}{j^2},$$

where

$$\alpha_j = \int\limits_0^1 u_1(x_1, x_2, x_3)\cos(\pi j x_3)\,dx_3, \quad \beta_j = \int\limits_0^1 u_2(x_1, x_2, x_3)\cos(\pi j x_3)\,dx_3$$

are coefficients of the Fourier series over a system of orthogonal functions $\sqrt{2}\cos(\pi j x_3)$, $j = 1, 2, \ldots$, for the velocity components u_1 and u_2.

As examples of the practical realization of the relationships proposed in this section we can offer the problems of mass transport in a cellular Bénard structure [13] or in a Taylor structure where the fluid flows between coaxial cylinders.

3. Dispersion of an Impurity in Nonsteady Fluid Flows. The scheme for the construction of dispersion equations and the equations themselves do not change if the velocity components are functions of time within the corresponding scale. For the case of a prismatic tube such a problem was dealt with in [14], where the author limited himself to an analysis of dispersion in a periodically variable velocity profile. Employing the moment approach [2] and then averaging over the period of the concentration moment, he obtained a value for the dispersion coefficient that was independent of time and which he proposed to use in the equation of equivalent diffusion

$$\partial c/\partial t = D^*\partial^2 c/\partial z^2. \tag{3.1}$$

In our opinion, it is more advisable to regard D^* as a function of time, as follows naturally in the utilization of the above-covered algorithm. For example, for a circular tube in a uniform flow, when the dependence on time is not necessarily periodic, we have

$$D^*(t) = 2\int_0^1 \frac{dr}{r}\left[\int_0^r \xi w(\xi, t)\, d\xi\right]^2, \tag{3.2}$$

where r is the radial coordinate of the cylindrical coordinate system. One would imagine that the fundamental significance of the theory of dispersion lies in the replacement of the complex equations of convective diffusion by substantially simpler equations with a smaller number of independent variables. These equations can be used (in determining the conditions of suitability) to analyze the mass-exchange processes in natural phenomena and in technical installations. In this connection, the construction of the equivalent equation of diffusion based exclusively on studies of the asymptotic properties of the first two moments

$$\mu_i = \int_{-\infty}^{+\infty} z^i c(x, y, z, t)\, dz, \quad M_i = \frac{1}{s}\int_\Omega \mu_i(x, y, t)\, dx\, dy, \quad i = 0, 1, 2,$$

as is frequently done in the literature, in particular in [14], is not convincing, since there is no assurance that no other (with the exception of $\partial^2 c/\partial z^2$) terms will enter the equation of dispersion, which would not "be noticed" by the moment approach. For example, the expression $\partial^3 c/\partial z^3$ under natural conditions of diminution in the concentration c and in its derivatives as $z \to \pm\infty$ yields zero in calculating the moments to the third order, inclusively:

$$\int_{-\infty}^{+\infty} z^i (\partial^3 c/\partial z^3)\, dz = 0, \quad i = 0, 1, 2,$$

i.e., for observation of such a term one should resort to moments of higher order, and the validated construction of Taylor-type equations in similar situations may become extremely complex.

In the case of a uniform flow in a tube the dependence on time on the part of the coefficient D^* involves no fundamental difficulties from the standpoint of analyzing the processes of impurity propagation on the basis of Eq. (3.1), since having introduced the "modified time" $t^* = \int_0^t D^*(\tau)\, d\tau$, we obtain an ordinary equation of diffusion in a coordinate system moving at an average velocity (along r), which in this case is dependent on time. Moreover, calculation of D^* according to (3.2) or (1.10), where the dependence on time w(x, y, t) is added to the velocity component w, but there is no dependence on z, so that this may turn out to be simpler than in accordance with the corresponding formulas from [14], while the dependence on t is rather general. In the case of nonuniform flows in elongated zones, the above-described reduction to an equation with a constant coefficient of diffusion is impossible and no significant simplifications appear even in the boundary-layer equations in the case of large Peclet numbers near the lines where w vanishes.

4. Some Remarks and Additions.

In the proposed equations of equivalent diffusion it is easy to include a number of effects which we had not taken into consideration. If the source of the substance of weak (on the order of ε^2) intensity within the volume or at the boundary surfaces is given, then in Eqs. (1.9) and (2.8) they will naturally be included as functions of c_0 and, possibly, as functions of other variables. An even more complex condition might prevail at the surface, such as, for example, $\partial c/\partial x_3|_{x_3=0} = \nu^2 g(c - c_*)$ (c_* is the equilibrium concentration, while the function g (on the order of unity) is arbitrary). We might also turn to coordinate systems that are nonmoving in space. In the case of Eq. (1.9), if we add to it a term of the form $\langle W\rangle \partial c/\partial z$, where the average velocity $\langle W\rangle$ is independent of the longitudinal coordinate, as follows out of the continuity equation, then in the case of Eq. (2.8) the components of the average velocity in the convective terms will depend on the coordinates

$$\frac{\partial c_0}{\partial x_1}\int_0^1 H_2H_3U_1\,dx_3+\frac{\partial c_0}{\partial x_2}\int_0^1 H_1H_3U_2\,dx_3$$

(the velocities W, U_1, and U_2 pertain to the nonmoving coordinate system). To calculate the coefficient of dispersion according to (1.10) and (2.9) it is necessary to know the distribution of certain velocity components within the region of flow. This separate problem is not dealt with here. However, let us note that we require the integral characteristics of velocity, which in a number of cases are determined more precisely than the local characteristics when using approximation methods.

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